RESEARCH STATEMENT

MARCO MAZZUCCHELLI

My primary research interests are in symplectic geometry, contact geometry and non-linear analysis, with particular emphasis toward the study of periodic orbits in Lagrangian and Hamiltonian dynamics. These subjects are at the crossroad of many disciplines, such as geometric analysis, algebraic and differential topology, and differential geometry. I am also interested in Aubry-Mather theory and weak KAM theory, especially in view of the recently arisen connections with symplectic geometry (see e.g. [PPS03, Ber07, Vit08, SV10, CFP10]). As an undergraduate student, I worked on the algebraic theory of inverse semigroups.

1. Summary of obtained results

1.1. **Results in Lagrangian dynamics.** In Lagrangian dynamics, the orbits of a system are characterized by a variational principle: they are critical points of the Lagrangian action functional. The study of existence and multiplicity of certain types of orbits (e.g. periodic orbits or, more generally, orbits with a prescribed boundary condition) can be carried out by applying global methods from critical point theory, for instance Morse theory, Lusternik-Schnirelmann theory, Conley theory and so forth, see e.g. [Cha93, Con78, BH04]. In order to apply these techniques, one needs to find a suitable setting for the action functional: a space of curves that has the structure of a (possibly infinite-dimensional) smooth manifold on which the functional is sufficiently smooth, say C^2, and has sufficiently compact (in a weak sense) sub-levels. In my Ph.D. thesis [Maz09], I considered Lagrangian functions that are fiber-wise convex with quadratic growth, and I developed a finite-dimensional functional setting for the associated Lagrangian action: in this setting, the domain of the action is given by continuous curves that are piece-wise solutions of the Euler-Lagrange equation. This may be seen as a generalization of Morse's broken geodesic approximation of the path space (see [Mil63]). In this setting, the action becomes a C^∞ function with compact sub-levels. By combining this with a modification technique developed by Abbondandolo and Figalli [AF07], I could deal with the more general class of Tonelli Lagrangians: these are fiber-wise convex and super-linear Lagrangians with global Euler-Lagrange flow. The Tonelli class is arguably the most important in Lagrangian dynamics: it is indeed the broadest class of coercive Lagrangians admitting a Hamiltonian dual. Moreover, it is the class considered in Aubry-Mather theory and weak KAM theory (see e.g. [Mat91, Fat08]).

As an application of the functional setting that I developed in my thesis, I could extend celebrated results by Long [Lon00] and Lu [Lu09], establishing a Lagrangian version of the so-called Conley conjecture for the full class of Tonelli Lagrangians: every non-autonomous

Last update: November 10, 2011.

and time-periodic Tonelli Lagrangian system on a closed configuration space admits infinitely many periodic orbits with uniformly bounded mean action. Moreover, I proved that either infinitely many of these orbits have the same basic period as the Lagrangian, or there are orbits with arbitrarily high period. These results appeared in [Maz11c].

The functional properties of the Lagrangian action are often overlooked in the literature. By means of my functional setting, I could provide a solid foundation and recover most of the proofs of published results affected by this problem. An extensive treatment is contained in my monograph [Maz11a], which will be published in December 2011.

1.2. **Results on billiards.** Billiard dynamics studies the motion of a point particle inside a compact domain of the Euclidean space which moves along straight lines with constant speed until it reaches the boundary of the domain, where it bounces with specular reflection and with no loss of energy. The study of periodic orbits in billiards began with Birkhoff [Bir27, Bir66] who investigated their existence and multiplicity when the billiard table is a convex compact subset of $\mathbb{R}^2$ with smooth boundary. In higher dimension, the same problem was addressed by Babenko [Bab90], Farber and Tabachnikov [FT02b, FT02a], Farber [Far02] and Karasev [Kar09]. Their results can be summarized by saying that the number of periodic billiard trajectories inside a convex region of $\mathbb{R}^N$ and whose number of bounce points divides a given odd $n \in \mathbb{N}$ grows at least linearly in n.

In my paper [Maz10], inspired by the work of Bott [Bot56] and Gromoll and Meyer [GM69b] on the closed geodesics problem, I developed an iteration theory for periodic billiard trajectories. More specifically, a periodic billiard trajectory bouncing n times is a critical point of the perimeter length functional ℓ^n defined on the space of n-gons inscribed in the billiard table. By applying Morse theory to the functional ℓ^n, one finds two possible types of critical points: the ones corresponding to closed billiard trajectories bouncing n times, and the ones corresponding to iteration of closed trajectories whose number of bounce points divides n. In my paper, I investigated the behavior of the Morse indices and of the local homology of billiard trajectories under iteration. As an application, I could establish a new multiplicity result for non-iterated periodic billiard trajectories with small Morse co-index with respect to the perimeter length functional.

For non-convex billiard tables, the perimeter length functional does not provide a good variational setting anymore: indeed, some of its critical points correspond to orbits that leave the table. In the 1980's, Benci and Giannoni provided a new approach for the periodic orbits problem in non-convex billiards. They considered systems where a particle moves inside the table according to a Hamiltonian flow, and then bounce at the boundary according to the usual bouncing rule. Their idea was to see this system as a limit of smooth Hamiltonian systems whose Hamiltonians blow-up close to the boundary of the configuration space. By using classical variational methods, they could prove the existence of periodic bounce orbits with a prescribed period and number of bounce points suitably bounded. In a recent joint work with Peter Albers [AM11], we combined the approach of Benci and Giannoni with techniques from symplectic geometry to obtain a dual statement in the same setting: the existence of periodic bounce orbits with prescribed energy and number of bounce points bounded by the dimension of the billiard table plus one. Both

Benci and Giannoni's and our statements imply the existence of one periodic orbit in any non-convex billiard with smooth boundary.

1.3. **Results on inverse semigroups.** Inverse semigroups are algebraic objects that generalize the notion of groups: they are semigroups S such that every $s \in S$ admits a unique generalized inverse $s^{-1} \in S$ with the property that $s = ss^{-1}s$ and $s^{-1} = s^{-1}ss^{-1}$. This notion has been introduced in the 1950's by Vagner [Vag52] and Preston [Pre54] (see also [Pet84]). In 1990, Stephen [Ste90] introduced presentation for inverse semigroups, establishing the foundations for a combinatorial theory in this category. In a joint paper with Alessandra Cherubini [MC08], we study amalgamated free products of inverse semigroups. Our main result asserts the decidability of the word problem for a broad class of finitely presented amalgamated free products under mild conditions on the amalgam.

2. Summary of ongoing and future projects

2.1. **The Conley conjecture and (non-)isolatedness of topologically degenerate orbits.** In 1984, Conley [Con84] conjectured that every Hamiltonian diffeomorphism of the standard symplectic torus has infinitely many periodic points. This celebrated conjecture has been recently established by Hingston [Hin09] and further extended to more general closed symplectic manifolds by Ginzburg and its collaborators [Gin10, GG10b, Hei09, GG10a]. Their arguments are based on a hard result asserting that, in a Hamiltonian system, a special type of periodic orbits, sometimes called topologically degenerate, have average action that is not-isolated in the set of average actions of periodic orbits. In an ongoing project, I wish to provide a new and more transparent proof of this result. Starting with the case of the torus, I wish to combine the machinery of generating functions with homological techniques developed by Bangert and Klingenberg [Ban80a, BK83], in order to prove the above statement. A preliminary version of my work is available at [Maz11b]. I believe that this approach may allow to prove that topologically degenerate periodic points are not isolated in the set of periodic points of a Hamiltonian diffeomorphisms. On the one hand, this would answer positively a question posed by Hingston in [Hin09]. On the other hand, this would allow to provide a new simple proof of the Conley conjecture also in more general closed symplectic manifolds.

A related project I am currently working on is devoted to the study of the isolatedness of topologically degenerate periodic orbits in Tonelli Lagrangian systems. Currently, by applying the results contained in my thesis, I could reduce this study to more fundamental questions about the maximal size of Gromoll-Meyer neighborhoods (see [GM69a, CG96]) of isolated critical points of abstract smooth functions. I believe that the results that I will obtain in this project may have independent interest, beyond the scope of Lagrangian dynamics.

2.2. **Low-energy periodic orbits in autonomous Lagrangian systems.** In his recent preprint [Con06], Contreras summarized all the known results about periodic orbits of autonomous Tonelli Lagrangian systems on closed configuration spaces. It is well known that Lagrangian dynamics in low energy levels, where low means below the so-called Mañé critical value, is extremely different from the one in high energies, which is conjugated to a Finsler geodesic dynamics. Currently, the best known result about periodic orbits with low energy is merely an existence one: for almost every energy level below the strict Mañé critical value there is at least one periodic orbit. In a joint project with Alberto Abbondandolo and Gonzalo Contreras we wish to study the multiplicity of periodic orbits in these low energy levels. The main difficulty in the project is the lack of a suitable setting for the Morse theory, as opposed to the high energy case where there is a suitable choice of loop space on which the Lagrangian action is smooth and has compact sub-levels (in the weak Palais-Smale sense). At this stage, our goal is to prove that any low energy level contains infinitely many periodic orbits provided it contains only hyperbolic periodic orbits or it contains at least a special, topologically degenerate, periodic orbit. Here, topologically degenerate has an analogous meaning as in the Hamiltonian situation, mentioned in section 2.1: an orbit is topologically degenerate when infinitely many of its iterations have non-trivial local homology (as critical point of the action functional, see [Mor96, GM69a, Cha93]) in a fixed degree.

2.3. **Leaf-wise intersections on energy hypersurfaces.** In a joint project with Will Merry we study the multiplicity of leafwise intersections in cotangent bundles. This is a variation of the periodic orbits problem, formulated in the late 1970s by Moser [Mos78] and further investigated by many authors [Ban80b, Hof90, EH89, Gin07, Dra08, Zil10, Gür10]. Given an exact symplectic manifold $(W, \mathrm{d}\lambda)$, a restricted contact-type hypersurface $(\Sigma, \lambda|_\Sigma)$, and a Hamiltonian diffeomorphism $\phi : W \to W$, a point of Σ that is mapped by ϕ into the same Reeb leaf of Σ is called leafwise intersection point. In their recent papers [AF10, AF08], Albers and Frauenfelder showed that leafwise intersections arise as critical points of a suitable perturbation of the Hamiltonian free-time action functional. For this perturbed functional, they developed a suitable version of Floer homology, called Rabinowitz-Floer homology. As an application, they established the existence of infinitely many leafwise intersections, provided W is a symplectically standard cotangent bundle over a suitable closed base space, Σ is a generic hypersurface, and ϕ is a generic diffeomorphism. If one removes the genericity assumption on ϕ, their proof fails: indeed, it may happen that the infinitely many critical points of the perturbed action that they find correspond to the same leafwise intersection lying on a closed Reeb leaf. In my ongoing project with Will Merry, we wish to prove that this is never the case. In order to reach this goal, we plain to introduce a localized version of Rabinowitz-Floer homology, and then to develop an iteration theory, analogous to the one for closed geodesics (see [Bot56, Maz11a]), in the setting of leafwise intersections.

In another joint project with Sheila Sandon, we study the multiplicity of translated points of contactomorphisms in contact Euclidean spaces. Translated points are a special class of leafwise intersections: in a contact manifold with cooriented contact structure (M, α), a point $x \in M$ is called translated by the contactomorphism $\psi : M \to M$ when it is mapped by ψ into the same Reeb leaf and $\psi^*\alpha$ is equal to α at x. If $(M, \alpha) = (J^1\mathbb{R}^d, \mathrm{d}z - y\,\mathrm{d}x)$ and ψ is a positive contactomorphism, i.e. it is positively transverse to the contact distribution, Sandon [San11] could employ spectral invariants techniques for generating functions (see

[Vit92, Bhu01, San10]) in order to assert that the set of iterated translated points of ψ, that is, the union of the translated points of all the iterated compositions of ψ, is infinite. By removing the positivity assumption on ψ, her proof fails: indeed, the translated points that she finds as critical points of associated generating functions may all correspond to a same fixed point of ψ. In our joint project we wish to remove this positivity assumption, in order to obtain a complete multiplicity result. By investigating the properties of the local homology of generating functions, we expect to be able to prove that any fixed point of ψ arising from a spectral invariant of a generating function for ψ will not arise from a spectral invariant of a generating function for ψ^k provided $k \in \mathbb{N}$ is sufficiently large.

2.4. **Iteration map in Floer homology.** In a joint project with Frol Zapolsky, we wish to construct and study the loop-iteration map in the context of Floer homology of cotangent bundles. A celebrated result due to Viterbo [Vit99] and reproved independently by Salamon and Weber [SW06] and Abbondandolo and Schwarz [AS06] asserts that the Floer homology of a time-periodic Hamiltonian defined on a symplectically standard cotangent bundle T^*M, where M is a closed manifold, is isomorphic to the singular homology of the free loop space ΛM. By means of this isomorphism, one can see the homology homomorphism induced by the loop nth-iteration map $\Psi^n : \Lambda M \to \Lambda M$ as a homomorphism between the Floer homology of a Hamiltonian H and the Floer homology of the Hamiltonian $H^{(n)}$ whose time-1 map is the time-n map of H. The goal of our project is to construct a Floer chain map inducing this homomorphism in Floer homology. Such a construction should then be extended to the action-filtered version of Floer homology, where we expect the map to have the same vanishing property of the analogous map in loop space homology (see [BK83, Maz11c, Maz11a]). Beside from being interesting on its own as an investigation of the functoriality properties of Floer homology, this construction would have an important application: it would allow to provide an alternative, simple, proof of Hein's result [Hei11] that establish the Conley conjecture for suitable Hamiltonians that are Tonelli at infinity in cotangent bundles. Moreover, our study may suggest an analogous construction for the Floer homology of closed symplectic manifolds, where there is no correspondence with loop space homology: this would provide a completely new approach to the results of Ginzburg and his collaborators [Gin10, GG10b, Hei09, GG10a] on the Conley conjecture in suitable closed symplectic manifolds.

2.5. **Euler-Lagrange billiards.** In a joint project with Peter Albers, we investigate a general notion of billiards, that we call Euler-Lagrange billiards, and the multiplicity of periodic orbits therein. Our systems are defined by a compact configuration space $M \subset \mathbb{R}^N$ with smooth boundary, and a Lagrangian $L : \mathrm{T}M \to \mathbb{R}$ that is C^1 along the zero section, while it is C^∞ and Tonelli outside. A suitable variational principle involving the free-time action functional provides a natural bouncing rule for these systems: at an impact time t, an orbit γ preserves its Lagrangian energy, and modifies its velocity so that $\mathrm{T}_{\gamma(t)}\partial M$ is the kernel of the covector $\partial_v L(\gamma(t), \dot\gamma(t^+)) - \partial_v L(\gamma(t), \dot\gamma(t^-))$. This bouncing rule reduces to the usual billiard one when the Lagrangian is a squared Riemannian norm, and it agrees with the one recently defined by Gutkin and Tabachnikov [GT02] when the Lagrangian is a squared Finsler norm. We expect to extend the validity of our result [AM11] described in section 1.2 to this general setting. This would provide, in particular, new results on periodic orbits in magnetic and Finsler billiards in non-convex configuration spaces.

References

[AF07] A. Abbondandolo and A. Figalli, *High action orbits for Tonelli Lagrangians and superlinear Hamiltonians on compact configuration spaces*, J. Differential Equations **234** (2007), no. 2, 626–653.

[AF08] P. Albers and U. Frauenfelder, *Infinitely many leaf-wise intersections on cotangent bundles*, preprint, `arXiv:0812.4426v2`, 2008.

[AF10] ———, *Leaf-wise intersections and Rabinowitz Floer homology*, J. Topol. Anal. **2** (2010), no. 1, 77–98.

[AM11] P. Albers and M. Mazzucchelli, *Periodic bounce orbits of prescribed energy*, Int. Math. Res. Not. (2011), no. 14, 3289–3314.

[AS06] A. Abbondandolo and M. Schwarz, *On the Floer homology of cotangent bundles*, Comm. Pure Appl. Math. **59** (2006), no. 2, 254–316.

[Bab90] I. K. Babenko, *Periodic trajectories of three-dimensional Birkhoff billiards*, Mat. Sb. **181** (1990), no. 9, 1155–1169.

[Ban80a] V. Bangert, *Closed geodesics on complete surfaces*, Math. Ann. **251** (1980), no. 1, 83–96.

[Ban80b] A. Banyaga, *On fixed points of symplectic maps*, Invent. Math. **56** (1980), no. 3, 215–229.

[Ber07] P. Bernard, *Symplectic aspects of Mather theory*, Duke Math. J. **136** (2007), no. 3, 401–420.

[BH04] A. Banyaga and D. Hurtubise, *Lectures on Morse homology*, Kluwer Texts in the Mathematical Sciences, vol. 29, Kluwer Academic Publishers Group, Dordrecht, 2004.

[Bhu01] M. Bhupal, *A partial order on the group of contactomorphisms of* $\mathbb{R}^{2n+1}$ *via generating functions*, Turkish J. Math. **25** (2001), no. 1, 125–135.

[Bir27] G. D. Birkhoff, *On the periodic motions of dynamical systems*, Acta Math. **50** (1927), no. 1, 359–379.

[Bir66] ———, *Dynamical systems*, With an addendum by Jurgen Moser. American Mathematical Society Colloquium Publications, Vol. IX, American Mathematical Society, Providence, R.I., 1966.

[BK83] V. Bangert and W. Klingenberg, *Homology generated by iterated closed geodesics*, Topology **22** (1983), no. 4, 379–388.

[Bot56] R. Bott, *On the iteration of closed geodesics and the Sturm intersection theory*, Comm. Pure Appl. Math. **9** (1956), 171–206.

[CFP10] K. Cieliebak, U. Frauenfelder, and G. P. Paternain, *Symplectic topology of Mañé's critical values*, Geom. Topol. **14** (2010), no. 3, 1765–1870.

[CG96] K. C. Chang and N. Ghoussoub, *The Conley index and the critical groups via an extension of Gromoll-Meyer theory*, Topol. Methods Nonlinear Anal. **7** (1996), no. 1, 77–93.

[Cha93] K.-C. Chang, *Infinite-dimensional Morse theory and multiple solution problems*, Progress in Nonlinear Differential Equations and their Applications, 6, Birkhäuser Boston Inc., Boston, MA, 1993.

[Con78] C. C. Conley, *Isolated invariant sets and the Morse index*, CBMS Regional Conference Series in Mathematics, vol. 38, American Mathematical Society, Providence, R.I., 1978.

[Con84] ———, *Lecture at the University of Wisconsin*, April 6, 1984.

[Con06] Gonzalo Contreras, *The Palais-Smale condition on contact type energy levels for convex Lagrangian systems*, Calc. Var. Partial Differ. Equ. **27** (2006), no. 3, 321–395.

[Dra08] D. L. Dragnev, *Symplectic rigidity, symplectic fixed points, and global perturbations of Hamiltonian systems*, Comm. Pure Appl. Math. **61** (2008), no. 3, 346–370.

[EH89] I. Ekeland and H. Hofer, *Two symplectic fixed-point theorems with applications to Hamiltonian dynamics*, J. Math. Pures Appl. (9) **68** (1989), no. 4, 467–489 (1990).

[Far02] M. Farber, *Topology of billiard problems. I, II*, Duke Math. J. **115** (2002), no. 3, 559–585, 587–621.

[Fat08] A. Fathi, *Weak KAM theorem in Lagrangian dynamics*, Cambridge Univ. Press, forthcoming, preliminary version number 10, 2008.

[FT02a] M. Farber and S. Tabachnikov, *Periodic trajectories in 3-dimensional convex billiards*, Manuscripta Math. **108** (2002), no. 4, 431–437.

[FT02b] ———, *Topology of cyclic configuration spaces and periodic trajectories of multi-dimensional billiards*, Topology **41** (2002), no. 3, 553–589.

[GG10a] V. L. Ginzburg and B. Z. Gürel, *Conley conjecture for negative monotone symplectic manifolds*, preprint `arXiv:1011.5018`, 2010.

[GG10b] ———, *Local Floer homology and the action gap*, J. Symplectic Geom. **8** (2010), no. 3, 323–357.

[Gin07] Viktor L. Ginzburg, *Coisotropic intersections*, Duke Math. J. **140** (2007), no. 1, 111–163.

[Gin10] V. L. Ginzburg, *The Conley conjecture*, Ann. Math. **172** (2010), no. 2, 1127–1180.

[GM69a] D. Gromoll and W. Meyer, *On differentiable functions with isolated critical points*, Topology **8** (1969), 361–369.

[GM69b] ———, *Periodic geodesics on compact Riemannian manifolds*, J. Differential Geometry **3** (1969), 493–510.

[GT02] E. Gutkin and S. Tabachnikov, *Billiards in Finsler and Minkowski geometries*, J. Geom. Phys. **40** (2002), no. 3-4, 277–301.

[Gür10] B. Z. Gürel, *Leafwise coisotropic intersections*, Int. Math. Res. Not. (2010), no. 5, 914–931.

[Hei09] D. Hein, *The Conley conjecture for irrational symplectic manifolds*, to appear in J. Symplectic Geom., 2009.

[Hei11] ———, *The Conley conjecture for the cotangent bundle*, Archiv der Mathematik **96** (2011), no. 1, 85–100.

[Hin09] N. Hingston, *Subharmonic solutions of Hamiltonian equations on tori*, Ann. Math. **170** (2009), no. 2, 529–560.

[Hof90] H. Hofer, *On the topological properties of symplectic maps*, Proc. Roy. Soc. Edinburgh Sect. A **115** (1990), no. 1-2, 25–38.

[Kar09] R. N. Karasev, *Periodic billiard trajectories in smooth convex bodies*, Geom. Funct. Anal. **19** (2009), no. 2, 423–428.

[Lon00] Y. Long, *Multiple periodic points of the Poincaré map of Lagrangian systems on tori*, Math. Z. **233** (2000), no. 3, 443–470.

[Lu09] G. Lu, *The Conley conjecture for Hamiltonian systems on the cotangent bundle and its analogue for Lagrangian systems*, J. Funct. Anal. **256** (2009), no. 9, 2967–3034.

[Mat91] J. N. Mather, *Action minimizing invariant measures for positive definite Lagrangian systems*, Math. Z. **207** (1991), no. 2, 169–207.

[Maz09] M. Mazzucchelli, *Periodic orbits of Tonelli Lagrangian systems*, Ph.D. thesis, Università di Pisa, 2009.

[Maz10] ———, *On the multiplicity of non-iterated periodic billiard trajectories*, Pacific J. Math. **252** (2010), no. 1, 181–205.

[Maz11a] ———, *Critical point theory for Lagrangian systems*, Progress in Mathematics, vol. 293, Birkhäuser Verlag, 2011.

[Maz11b] ———, *A discrete approach to the Conley conjecture for tori*, preliminary version `arXiv:1102.2021`, 2011.

[Maz11c] ———, *The Lagrangian Conley conjecture*, Comment. Math. Helv. **86** (2011), no. 1, 189–246.

[MC08] M. Mazzucchelli and A. Cherubini, *On the decidability of the word problem for amalgamated free products of inverse semigroups*, Semigroup Forum **76** (2008), 309–329.

[Mil63] J. Milnor, *Morse theory*, Based on lecture notes by M. Spivak and R. Wells. Annals of Mathematics Studies, No. 51, Princeton University Press, Princeton, N.J., 1963.

[Mor96] M. Morse, *The calculus of variations in the large*, American Mathematical Society Colloquium Publications, vol. 18, American Mathematical Society, Providence, RI, 1996, Reprint of the 1932 original.

[Mos78] J. Moser, *A fixed point theorem in symplectic geometry*, Acta Math. **141** (1978), no. 1–2, 17–34.

[Pet84] M. Petrich, *Inverse semigroups*, Pure and Applied Mathematics (New York), John Wiley & Sons Inc., New York, 1984.

[PPS03] G. P. Paternain, L. Polterovich, and K. F. Siburg, *Boundary rigidity for Lagrangian submanifolds, non-removable intersections, and Aubry-Mather theory*, Mosc. Math. J. **3** (2003), no. 2, 593–619.

[Pre54] G. B. Preston, *Inverse semi-groups*, J. London Math. Soc. **29** (1954), 396–403.

[San10] S. Sandon, *An integer-valued bi-invariant metric on the group of contactomorphisms of* $\mathbb{R}^{2n} \times S^1$, J. Topol. Anal. **2** (2010), no. 3, 327–339.

[San11] ———, *On periodic translated points for contactomorphisms of* $\mathbb{R}^{2n+1}$ *and* $\mathbb{R}^{2n} \times S^1$, preprint `arXiv:1102.4202`, 2011.

[Ste90] J. B. Stephen, *Presentations of inverse monoids*, J. Pure Appl. Algebra **63** (1990), 81–112.

[SV10] A. Sorrentino and C. Viterbo, *Action minimizing properties and distances on the group of Hamiltonian diffeomorphisms*, Geom. Topol. **14** (2010), no. 4, 2383–2403.

[SW06] D. A. Salamon and J. Weber, *Floer homology and the heat flow*, Geom. Funct. Anal. **16** (2006), no. 5, 1050–1138.

[Vag52] V. V. Vagner, *Generalized groups*, Doklady Akad. Nauk SSSR (N.S.) **84** (1952), 1119–1122.

[Vit92] C. Viterbo, *Symplectic topology as the geometry of generating functions*, Math. Ann. **292** (1992), no. 4, 685–710.

[Vit99] ———, *Functors and computations in Floer homology with applications. I*, Geom. Funct. Anal. **9** (1999), no. 5, 985–1033.

[Vit08] ———, *Symplectic homogenization*, preprint `arXiv:0801.0206`, 01 2008.

[Zil10] Fabian Ziltener, *Coisotropic submanifolds, leaf-wise fixed points, and presymplectic embeddings*, J. Symplectic Geom. **8** (2010), no. 1, 95–118.

Department of Mathematics, Pennsylvania State University, USA
E-mail address: `mazzucchel@math.psu.edu`